Deutsch algorithm and decoherence

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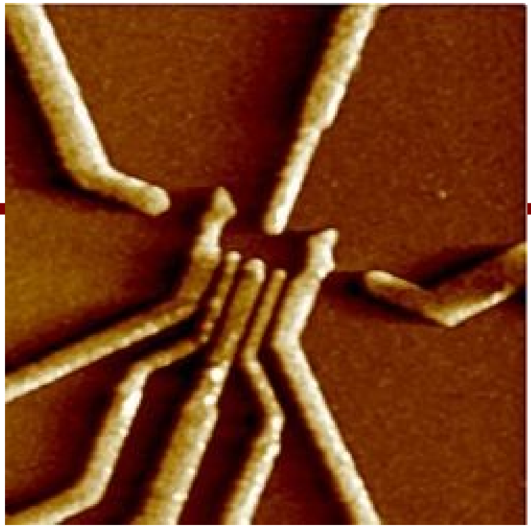
Quantum computing
The original idea



Bits → Qbits
0, 1 → $|0\rangle, |1\rangle$

superposition
$\alpha|0\rangle + \beta|1\rangle$

gates → quantum gates (unitary operators)

=> algorithms can “act” on both states at the same time

The relation between $|0\rangle$ and $|1\rangle$ can be made use of.

Example: Deutsch algorithm

The problem: The distinction between a constant and a balanced function (here – for one bit input and output).

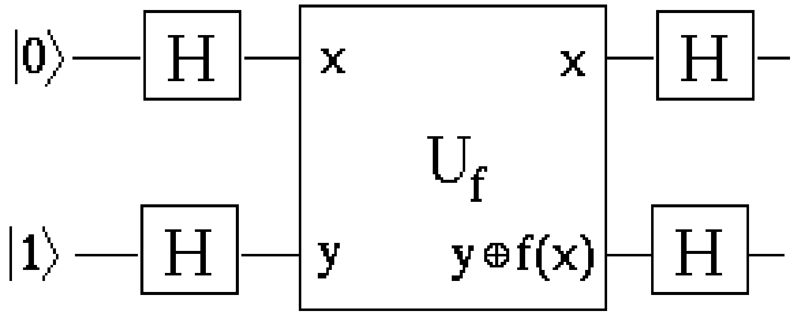
Constant:

$$\begin{cases} f(0) = 0 \\ f(1) = 0 \end{cases} \qquad \begin{cases} f(0) = 1 \\ f(1) = 1 \end{cases}$$

Balanced:

$$\begin{cases} f(0) = 0 \\ f(1) = 1 \end{cases} \qquad \begin{cases} f(0) = 1 \\ f(1) = 0 \end{cases}$$

Classically the function needs to be found twice, to find f(0) and f(1);
a quantum computer could do it in one go.



Hadamard gate

$$U_H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Implementation of the function f (two-qubit gate)

$$U_f : |x\rangle_A |y\rangle_B \to |x\rangle_A |y \oplus f(x)\rangle_B$$

$$U_f |x\rangle_A (|0\rangle - |1\rangle)_B = (-1)^{f(x)} |x\rangle_A (|0\rangle - |1\rangle)_B$$

$$|0\rangle_A \otimes |1\rangle_B \xrightarrow{H} \frac{1}{2}(|0\rangle + |1\rangle)_A \otimes (|0\rangle - |1\rangle)_B \xrightarrow{f} \frac{1}{2}(|0\rangle + (-1)^{f(0)\oplus f(1)}|1\rangle)_A \otimes (|0\rangle - |1\rangle)_B$$

$$\xrightarrow{H} \frac{1}{2}\left[\left(1 + (-1)^{f(0)\oplus f(1)}\right)|0\rangle + \left(1 - (-1)^{f(0)\oplus f(1)}\right)|1\rangle\right]_A |1\rangle_B$$

Measurement of qubit A yields $|0\rangle$ for constant and $|1\rangle$ for balanced functions f.

Physical implementation
Problems

→ quantum control
→ scalability
→ decoherence
Most qubits are not isolated systems.

Decoherence from classical sources (charge noise)
Decoherence from quantum sources: entangling, non-entangling.

Qubit-environment entanglement => operations and measurements on qubits also affect the environment.